QUARK MASSES AND THE STRONG COUPLING FROM LATTICE QCD

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OUTLINE

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- Results
  - $\alpha_s(m_Z)$
  - light quark masses
  - heavy quark masses
INTRODUCTION
QCD LAGRANGIAN

- Aim: determine the fundamental parameters of QCD

\[ \mathcal{L}_{\text{QCD}} = \frac{1}{4g_0^2} F_{\mu\nu}^a F^{a\mu\nu} + \sum_f \bar{q}_f (\mathbb{D} + m_{0f}) q \]

- Calculate \( n_f + 1 \) observables.

- Adjust \( g_0 \) and \( m_{0f} \) so that they agree with \( n_f + 1 \) experimental measurements.
• Bare coupling and bare masses depend on UV cutoff: prefer renormalized coupling and mass.

• Standard renormalization scheme(s) facilitate communication: low-energy QCD is an initial condition for high-energy GUTs.

• No QCD calculation is full and exact, so need $n_f + 1 + \text{many calculations to validate choices.}$
In QFT, the standard is the modified minimal subtraction scheme, $\overline{\text{MS}}$.

Necessarily perturbative: a disadvantage when it is possible to relate the Lagrangian to experiment nonperturbatively.

Still: I will have to discuss the relationship between $\{\alpha_0(a), m_0f(a)\}$ and $\{\bar{\alpha}_s(\mu), \bar{m}_f(\mu)\}$. 
In lattice QCD, the UV cutoff is the (inverse) lattice spacing, $a^{-1}$.

Calculate hadron properties from functional integral: Monte Carlo; importance sampling.

Biggest computational challenge: including sea of up, down, and strange quarks.

Staggered, Wilson, twisted, domain-wall, overlap.
The uncertainties are smallest for a class of quantities we call **gold-plated**.

- Stable hadrons, well-below threshold.
- Their masses, and matrix elements with zero or one such hadron in initial-final state.
- “Chiral extrapolation” under control.
Numerical lattice QCD should at least be able to get gold-plated quantities correct, before using it to determine $\alpha_s$ and $m_q$.

Otherwise errors are not quantifiable.

Only the fastest method for sea quarks has achieved this; others are 5–10 years behind (with 3–4 passed).
Davies et al., hep-lat/0304004

**STAGGERED SEA**

![Graph showing comparison between quenched/experiment and (n_f = 2+1)/experiment](image_url)

- $f_\pi$
- $f_K$
- $3 m_\Xi - m_N$
- $2 m_B - m_Y$
- $\psi(1P-1S)$
- $Y(1D-1S)$
- $Y(2P-1S)$
- $Y(3S-1S)$
- $Y(1P-1S)$

The graph compares the results of quenched and experiment settings with those from the sea with $(n_f = 2+1)$.
Using lattice QCD with staggered quarks, we have had several predictions confirmed:

- mass of the $B_c$ meson
- decay constants $f_D$ and $f_{Ds}$
- $q^2$ dependence and normalization of $D \rightarrow Kl\nu$ form factors
The $\Upsilon(1S)$ mass determines $m_b$.

The $\eta_c-J/\psi(1S)$ mass determines $m_c$.

In the ratio plot, the lattice spacing $a$ comes from the $\Upsilon(2S)-\Upsilon(1S)$ mass difference;

any other quantity in the plot would, in retrospect, do as well.
• Setting $a$ lends meaning to the scale $\mu$ in $\alpha_s(\mu)$.

• As an intermediate step, many modern lattice calculations are converted to $r_0$ or $r_1$ units:

$$r_0^2 F(r_0) = 1.65, \quad r_1^2 F(r_1) = 1.00,$$

where $F$ is the force between static sources.

• The $\Upsilon$ splittings yield $r_0$ & $r_1$ in physical units.
Methods 1: Strong Coupling
Compute, say, scattering of quarks at high energy) in lattice PT and continuum PT, and demand that they’re equal.

Matching condition:

\[ \tilde{\alpha}_s(a^{-1}) = \frac{g_0^2}{4\pi} + c_1 \left( \frac{g_0^2}{4\pi} \right)^2 + c_2 \left( \frac{g_0^2}{4\pi} \right)^4 + \cdots \]

The \( c_i \) are HUGE. This is a disaster!
• The hugeness comes from Feynman diagrams that arise in lattice PT but are absent in continuum PT.

• This appears in all expansions in $g_0$

• Lattice perturbation theory becomes normal if the series is rearranged to get rid of $g_0$ [Lepage, Mackenzie].
• For example, take the $1 \times 1$ Wilson loop:

$$\ln W_{11} = c_0 \frac{g_0^2}{4\pi} + c_1 \left( \frac{g_0^2}{4\pi} \right)^2 + c_2 \left( \frac{g_0^2}{4\pi} \right)^4 + \cdots$$

• Define $\alpha_{1 \times 1} = \ln W_{11}/c_0$

• Expansions in $\alpha_{1 \times 1}$ behave normally (for perturbative QCD), e.g., that for $\alpha_{\overline{\text{MS}}}$
- So, a strategy for determining is to analyze lots of short distance quantities perturbatively.

- The key is to express expansions in $\alpha_{\text{MS}}(\mu)$ or some other scheme such as $\alpha_{V}(\mu)$.

- This analysis (now multi-loop) finds that PT works well, especially with a BLM choice of the scale $\mu$. 
One can also obtain the $\Lambda$ parameter:

$$\Lambda = M e^{-\frac{4\pi}{2\beta_0 \alpha_s(M)} \left(4\pi \beta_0 \alpha_s\right)^{-\frac{1}{2}\beta_1/2\beta_0^2}} \times \exp \left[-\frac{1}{2} \int_0^{\alpha_s(M)} 3 \text{ loop etc.} \right]$$

Often done with a nonperturbative version of $\alpha_s(M)$, but perturbation theory still needed for $\beta$ function.
METHODS II:
LIGHT MASSES
The pole mass is IR finite and gauge invariant with any UV regulator, so it can be used for matching:

\[
\begin{align*}
    m_{\text{pole}} &= Z_{m_0} m_0 a \, a^{-1} \\
    m_{\text{pole}} &= Z_{\bar{m}} \bar{m}(\mu)
\end{align*}
\]

\[
\bar{m}(\mu) = \frac{Z_{m_0}}{Z_{\bar{m}}} m_0 a \, a^{-1}
\]

In \(Z_{m_0}/Z_{\bar{m}}\) IR renormalons cancel.
One can also obtain the RGI mass:

\[ m_{\text{RGI}} = m(M) \left[ \frac{\alpha_s(M)}{4\pi} \right]^{\gamma_0/\beta_0} \exp \left[ \int_0^\infty \frac{\alpha_s(M)}{2} \text{loop etc.} \right] \]

Often done with a nonperturbative version of the renormalized mass \( m(M) \), but perturbation theory still needed for anomalous dimension.
METHODS III: HEAVY MASSES
NEW STRATEGY

- The Compton wavelength of charmed and bottom quarks is similar to the lattice spacing.
  - $a = 0.06–0.18$ fm
  - $m_c a = 0.35–1.0$
  - $m_b a = 1.4–4.2$

- So they can/should/must be treated as short distances with effective field theories.
LONG STORY MADE SHORT

- $\mathcal{L}_{HQ} = -m_1 \bar{Q}Q + \frac{\bar{Q}D^2Q}{2m_2} + \cdots$
- Arrange things so the only large consequence of this is in the rest mass, $m_1$.
- Quark masses tuned to kinetic mass, $m_2$.
- Other effects of lattice lumped into small effects, of higher order in NRQCD or HQET power counting.
Radiative corrections

- The rest mass acquires an additive term.
- The kinetic mass acquires a multiplicative factor.
Define binding energy $B = M_1 - 2m_1$, with some regulator-independent, renormalized mass, like the pole mass or PS mass, ....

Then, e.g., $m_{PS}(\mu_f) = \frac{M_{\text{expt}} - B_{PS}(\mu_f)}{2}$.

Convert to $\bar{m}(\mu)$
Formulas look like light-quark matching:

\[ m_{\text{pole}} = Z_{m_2} m_2 a \frac{M_{\text{expt}}}{M_2 a} \]

\[ m_{\text{pole}} = Z \bar{m} \bar{m}(\mu) \]

but the last factor reduces tuning uncertainty.

Once again, renormalons cancel.
RESULTS
Results from the quenched approximation, or with two strangish sea quarks, are discarded.

Results with 2+1 flavors of sea quarks, with Wilson, twisted, domain-wall, overlap do not yet exist.

Therefore, present results with 2+1 staggered quarks.
Mason et al. [hep-lat/0503005] compute many short-distance quantities in MC, and analyze with 3-loop PT & 4-loop $\beta$.

- Static self-energy; cusps; gluon surfaces; gluon strings.
- They find

- $\alpha_V^{(3)}(7.5 \text{ GeV}) = 0.2082(40)$

- $\alpha_{\overline{\text{MS}}}^{(5)}(M_Z) = 0.1170(12)$

- The dominant uncertainties are, first, the determination of $\alpha$, and, second, the truncation of PT.
Test of Running

Mason et al. [hep-lat/0503005]
Light Quark Masses

- HPQCD + MILC find
  - $m_s(2\text{ GeV}) = 76(0)(3)(0)(7)\text{ MeV}$ hep-lat/0405022
  - $2m_s/(m_u + m_d) = 27.4(1)(4)(1)$ hep-lat/0405022
  - $m_u/m_d = 0.43(0)(1)(8)$ hep-lat/0407028

- There is an update, now with 2-loop matching
  - $m_s(2\text{ GeV}) = 87(0)(4)(0)(4)\text{ MeV}$ hep-lat/0511160
The $\Upsilon$ spectrum of Gray et al. [hep-lat/0507013] yields

- $r_0 = 0.469(7)$ fm
- $r_1 = 0.321(5)$ fm
- $\bar{m}_b(\bar{m}_b) = 4.4(3)$ GeV
CONCLUSIONS
Lattice QCD with staggered sea quarks provides

- the most accurate light quark masses
- bottom and charm masses that will be competitive when 2-loop matching is done
- (one of) the most accurate values of $\alpha_{\overline{MS}}$