Reconciling the Light-Cone and NRQCD Approaches to Calculating $e^+e^- \rightarrow J/\psi + \eta_c$

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Outline

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The Conflict Between Theory and Experiment

- **Experiment**
  - **Belle:** \( \sigma(e^+e^- \rightarrow J/\psi + \eta_c) \times B_{>2} = 25.6 \pm 2.8 \pm 3.4 \text{ fb} \).
  - **BaBar:** \( \sigma(e^+e^- \rightarrow J/\psi + \eta_c) \times B_{>2} = 17.6 \pm 2.8 \pm 2.1 \text{ fb} \).

- **NRQCD at LO in** \( \alpha_s \) **and** \( v $$
  - **Braaten, Lee:** \( \sigma(e^+e^- \rightarrow J/\psi + \eta_c) = 3.78 \pm 1.26 \text{ fb} \).
  - **Liu, He, Chao:** \( \sigma(e^+e^- \rightarrow J/\psi + \eta_c) = 5.5 \text{ fb} \).
  - The two calculations employ different choices of \( m_c \), NRQCD matrix elements, and \( \alpha_s \). Braaten and Lee include QED effects.

- **Trends**
  - The Belle cross section has moved down from \( 33_{-6}^{+7} \pm 9 \text{ fb} \).
  - The BaBar cross section is even lower.
  - Braaten and Lee found a sign error in the QED interference term that raised the prediction from \( 2.31 \pm 1.09 \text{ fb} \).
  - Zhang, Gao, Chao: A calculation of corrections at NLO in \( \alpha_s \) shows that the \( K \) factor may be as large as 1.8.
  - The trends are in the right direction, but a discrepancy remains.
A Proposed Solution

- Bondar and Chernyak (BC) and Ma and Si have proposed calculating the cross section in the light-cone formalism.
  - Focus on the BC calculation.

- BC find
  \[ \sigma(e^+e^- \rightarrow J/\psi + \eta_c) = 33 \text{ fb}. \]
  - They attribute the increased cross section to the finite width of the light-cone distribution.
  - Takes into account corrections that arise from the relative momentum of the \( Q \) and \( \bar{Q} \) in the quarkonium.
  - If the light-cone distribution is replaced by \( \delta(z - 1/2) \), the cross section reduces to \( \approx 3 \text{ fb} \).

- The NRQCD and light-cone approaches are both believed to be valid approximations to QCD in the limit \( E >> m_c, \Lambda_{QCD} \).

- Why is there such a large discrepancy between the NRQCD and light-cone results?
Possible Reasons for the Apparent Discrepancy Between the Light-Cone and NRQCD Calculations

BC Used *Ad Hoc* Model Quarkonium Light-Cone Distributions.

- The model light-cone distributions may not be the true light-cone distributions of QCD.
- We derived a light-cone distribution from potential-model wave functions (Cornell potential).
- If the model potential is the exact static $Q\bar{Q}$ potential, then the errors in the potential model are of relative order $v^2$.
  - $v$ is the velocity of the $Q$ or $\bar{Q}$ in the quarkonium rest frame.
  - $v^2 \approx 0.3$ for charmonium.
- The Cornell potential provides a good fit to lattice data for the static $Q\bar{Q}$ potential.

We computed the wave function by numerical integration of the Schrödinger Equation.
Calculate the Light-Cone Distribution $\phi(z)$

- Fourier transform the wave function:
  $$\psi(x) \rightarrow \psi(p).$$

- Use on-shell kinematics to relate $p$ to the light-cone momentum fraction:
  $$z = \frac{p^+}{P^+} = \frac{E_Q + p_3}{2E_Q}, \quad E_Q = \sqrt{p^2 + m_c^2}.$$  
  Error of relative order $v^2$.

- Integrate out $p_\perp$ to form the light-cone distribution $\phi(z)$.

- The integrals converge slowly, and tricks are needed to achieve adequate numerical accuracy. Checked against analytic forms in special cases.
Conclusions:

- The BC light-cone distribution agrees approximately with the shape of potential-model light-cone distribution.
- The BC and potential-model light-cone distributions give similar results for the cross section.
The High-Momentum Tail of the Light-cone Distribution Requires Special Treatment

- The regions near $z = 0$ and $z = 1$ correspond to large $p^2$.
- The potential-model wave function is not valid for $p^2 \gtrsim m_c^2$.
- In NRQCD calculations, contributions from the high-momentum tail of the wave function are contained in the short-distance coefficients.
  - Can use asymptotic freedom to compute them in perturbation theory.
  - They first appear at one-loop order (relative order $\alpha_s$).
- We subtract these contributions from the high-momentum regions of the light-cone distribution.
  - The remainder of the light-cone distribution is described accurately by the potential model.
  - The subtraction would be necessary even if the high-momentum tails of the light-cone distribution were accurate.
  - Otherwise, the light-cone calculation would contain part of the order-$\alpha_s$ NRQCD calculation and couldn’t be compared with the leading-order NRQCD calculation.
• The subtraction $\Delta \psi$ can be computed from the NRQCD matching condition:

$$\int (dp) \Delta \psi(p) H(p) = \begin{array}{c}
\begin{array}{c}
\hline
k \quad p \\
\hline
\end{array}
\end{array} - \begin{array}{c}
\begin{array}{c}
\hline
-k \\
\hline
\end{array}
\end{array}$$

• $\Delta \phi$ agrees asymptotically (near $z = 0, 1$) with $\phi$. The Coulomb part of the potential dominates in this region.

• $\Delta \phi$ appears to be singular at $z = 1/2$, but is actually a $+\Delta$ distribution satisfying

$$\int_0^1 dz \Delta \phi(z) = 0.$$

• Subtraction of $\Delta \phi$ shifts contributions from $z \approx 0, 1$ to $z \approx 1/2$.

• Subtraction of $\Delta \phi$ reduces the cross section by about a factor of 3.
BC Include Some Factors $Z_i$.

- The $Z_i$ correspond to renormalization of $m_c$ and the point-like tensor and pseudo-scalar vertices.
- They have no counterpart in the NRQCD calculations.
- They are included in an attempt to resum large logarithms of the momentum transfer divided by $m_c^2$.
- At least some of the $Z_i$ factors seem to be inappropriate.
  - The vertex-renormalization $Z_i$'s account correctly only for the evolution of the first moments of the light-cone distributions.
  - In BC all of the factors of $m_c$ are evolved with $Z_i$ factors.
    But some of the factors of $m_c$ correspond to $M_{\text{meson}} \approx 2m_c$ and have nothing to do with the running of $m_c$ in off-shell propagators.
- When the $Z_i$ are set to unity, the cross section is reduced by about a further factor of 2.
Numerical Results

<table>
<thead>
<tr>
<th>$Z_i$</th>
<th>BC</th>
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</thead>
<tbody>
<tr>
<td>$\alpha_s$</td>
<td>BC</td>
<td>BC</td>
<td>0.21</td>
<td>0.21</td>
</tr>
<tr>
<td>$m_c$ (GeV)</td>
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<td>1.2</td>
<td>1.2</td>
<td>1.4</td>
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<td>$\sigma_0$ (fb)</td>
<td>22.51</td>
<td>11.59</td>
<td>7.58</td>
<td>4.95</td>
</tr>
<tr>
<td>$\sigma$ (fb)</td>
<td>7.77</td>
<td>4.30</td>
<td>3.29</td>
<td>2.48</td>
</tr>
<tr>
<td>$\sigma_\delta$ (fb)</td>
<td>5.39</td>
<td>3.04</td>
<td>2.48</td>
<td>1.95</td>
</tr>
</tbody>
</table>

- $\sigma_0$: unsubtracted light-cone cross section (neglecting light-cone distributions of higher order in $v$).
- $\sigma$: light-cone cross section after $\Delta\phi$ has been subtracted.
- $\sigma_\delta$: light-cone cross section with a zero-width light-cone distribution: $\phi(z) \sim \delta(z - 1/2)$.
- BC allow $\alpha_s$ to run inside the integrations over $z$ and take $m_c = 1.2$ GeV.
- $\alpha_s = 0.21$ and $m_c = 1.4$ GeV are the values used in the Braaten-Lee NRQCD calculation.

- The light-cone cross section, after subtraction of $\Delta\phi$ and with $Z_i = 1$, is comparable to the LO NRQCD cross sections.
- The difference between $\sigma$ and $\sigma_\delta$ is small ($\approx 27\%$): After subtraction of $\Delta\phi$, the finite width of the light-cone distribution does not produce a large effect.
• In NRQCD, the corrections of higher order in $v$ to the cross section are known to be large: a factor of approximately 2.44.

• Why is this not reflected in the difference between $\sigma$ and $\sigma_\delta$?

• Of the large factor, approximately 1.82 comes from effects of higher order in $v$ in $\Gamma(J/\psi \rightarrow l^+ l^-)$.
  – Used to determine the wave function at the origin in the NRQCD calculation.
  – BC do not include the corresponding contribution in their light-cone calculation.

• The remaining factor in the NRQCD calculation is approximately 1.34.
  – The light-cone calculation does not fully reproduce this factor because it neglects $p_\perp$ in comparison with $p_3$.
  – However, these contributions are suppressed as $M_{\text{meson}}^2 / E_{\text{beam}}^2 \approx 0.34$, which makes the effect small.
Conclusions

- Once the high-momentum contributions are subtracted and the $Z_i$ are set to unity, the light-cone calculation is compatible with the NRQCD calculations.
- The BC light-cone calculation does not resolve the discrepancy between theory and experiment.
- Most promising theoretical avenues to resolve this discrepancy:
  - corrections of higher order in $\alpha_s$,
  - corrections of higher order in $v$.
- See the talk by Jungil Lee.