Quarkonium production in AA collisions and CGC

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Heavy Quark Production in pA collisions

- Coherence length of qq pair:
  \[ l_c \approx \frac{E_q}{(2m_q)^2} = \frac{1}{2M_N x_2} \]

  Where
  \[ x_2 = \frac{m_T}{\sqrt{s}} e^{-y} \]

- For charm at RHIC:
  \[ l_c = 15 e^y \text{ fm} \]

- At forward rapidity:
  \[ l_c \gg R_A \]

- Dipole model: color dipoles diagonalize the scattering matrix.
Quasi-classical picture of pA collisions

Quasi-classical approximation: no evolution

Quasiclassical-approximation: \( \alpha_s \ln(1/x) \ll 1 \quad \alpha_s^2 A^{1/3} \sim 1 \)

\[
\text{Im}N(x, r, b) = 1 - \exp \left( -\frac{r^2 Q_s^2}{4} \right)
\]

The dipole-nucleus cross section

\[
\sigma_{q\bar{q}} = 2 \int d^2b \text{Im}N(x, r, b)
\]

A. Mueller, McLerran-Venugopalan

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Time sequence

$A$

$p \rightarrow q \rightarrow \bar{q}$
Time sequence

\[ p \rightarrow q \rightarrow \Psi_1 \rightarrow 1 \]
Time sequence
Example of a dipole-nucleus propagator

\[-P(x, x_0) = -\frac{1}{8} x^2 Q_s^2 - \frac{1}{8} x_0^2 Q_s^2 + \frac{1}{8N_c^2} (x - x_0)^2 Q_s^2 \Rightarrow e^P\]

\[Q_s^2 = \frac{2\pi^2 \alpha_s N_c}{C_F} \rho T(b) x G(x, 1/x^2)\]

see also
Kopeliovich, Tarasov

K.T. hep-ph/0401022
Single inclusive quark cross-section

\[
\frac{d\sigma}{d^2kdy_1dy_2}(z_{01}) = \frac{1}{2(2\pi)^4} \int d^2z'_0 d^2z'_1 n_1(z_0, z_1; z'_0, z'_1; Y - y) d^2x_1 d^2x_2 d^2y_1 e^{-ik \cdot (x_1 - y_1)}
\]

\[
\times \int_0^1 d\alpha \sum_{i,j=1}^3 \sum_{k,l=0}^1 (-1)^{k+l} \Phi_{ij}(x_1 - z_k, x_2 - z_k; y_1 - z_l, x_2 - z_l; \alpha) \Xi_{ij}(x_1, x_2, z_k; y_1, x_2, z_l; \alpha, y)
\]

Size of the initial dipole

Number of dipoles in proton

Total rapidity Quark rapidity

qq wave functions for different time orderings

Rescattering factors

see also Blaizot, Gelis, Venugopal hep-ph/0402257

KT hep-ph/0401022
Kovchegov, KT hep-ph/0603055
At high enough energies/rapidities a quasiclassical approximation breaks down: $\alpha_s \ln(1/x) \sim 1$, $\alpha_s^2 A^{1/3} \sim 1$.

This requires resummation of multiple soft (x<<1) gluon emission in addition to multiple rescatterings.
Beyond a quasi-classical approximation

This resummation amounts to replacement of the Glauber exponent by a solution of the BK evolution equation (Kovchegov, K.T. hep-ph/0111362):

\[ e^{-\frac{1}{4}(x_0-x_1)^2Q_s^2\ln(1/|x_0-x_1|\mu)} \rightarrow N(x_0, x_1, Y) \]

\[ \partial_y N(x_{01}) = \frac{\alpha_s}{2\pi} \int d^2x_2 \frac{x_{01}^2}{x_{12}^2 x_{20}^2} (N(x_{12}) + N(x_{20}) - N(x_{01}) - N(x_{12})N(x_{20})) \]

The saturation scale acquires the energy dependence

\[ Q_s^2 = \Lambda^2 A^{1/3} e^{\lambda y} s^{\lambda/2} \]

Number of dipoles in proton (deuteron) evolves according to the BFKL equation.

Balitski-Kovchegov
also JIMWLK
Background field method

Structure of the partonic cascade at high energies:

\[ \tau = l_c = \frac{p_+}{m^2_\perp} \]

Strength of the field \( E \) is determined by the density of color charges in the transverse plane, i.e. \( Q_s(y) \)
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Field configuration at low $x$

(Chromo) Electric and Magnetic fields of a high energy hadron/nucleus are transverse (plane wave).

However, this is true only for a non-interacting hadron. Boundary conditions at the interaction point generate the **longitudinal** fields (Kharzeev, KT hep-ph/051234; Kharzeev, Levin, KT hep-ph/0602063)

$$A_+(x_-, x_+, x_\perp) = 0, \quad x_+ \leq x_-$$

It can be shown that in the LLA

$$E_\parallel \gg E_\perp \simeq B_\perp \gg B_\parallel$$

see also Fries, Kapusta, Li hep-ph/0511101 and by Lappi, McLerran hep-ph/0602189
Non-perturbative pair production

- The work done by the external chromo-electric field $E$ accelerating a virtual $qq$ pair apart by a Compton wavelength $\lambda_c = \hbar/mc$ is $W = gE\hbar/mc$.

- If $W > 2mc^2$ the pair becomes real.

\[
E_{cr} = \frac{m^2 c^3}{g\hbar}
\]

- In QED $E_{cr} = 10^{16}$ V/cm - beyond the current lab frontier.

- In QCD $g \sim 1$, $m \sim 1$ GeV, thus $E_{cr} \sim (1 \text{ GeV})^2$: pair production is a common phenomenon.
Propagator in external field

Pair production rate = imaginary part of the propagator

WKB approximation is quite useful for calculating tunneling probabilities.

Gelis, Kajantie, Lappi: numerical solution of Dirac equation in external field, hep-ph/0409058
WKB approximation

In the pair production process electron’s energy changes from $\varepsilon_-$ to $\varepsilon_+$ where

$$\varepsilon_{\pm} = \pm \sqrt{p^2(z) + m^2 + eEz}$$

Pair production rate $\Gamma \approx e^{-2\text{Im}S}$. Imaginary part of action $\text{Im}S$ can be found by integrating phase over states with imaginary momentum:

$$\Gamma = \exp\left\{-\int_{z_a}^{z_b} dz \left| p(z) \right| \right\} = \exp\left\{-\pi m^2_\perp / gE \right\}$$

where the turning points of the linear potential are

$$z_{a,b} = (\varepsilon \pm m) / gE$$
Time evolution of pair production

At early proper time $\tau \ll 1/Q_s$ the field $E = \text{const.}$

$$\phi(p_\perp) \propto S_\perp e^{-\frac{2\pi m^2_\perp}{gE}} = S_\perp e^{-\frac{p_\perp^2}{Q_s^2}}$$

At later time $\tau \sim 1/Q_s$ the field $E \propto e^{-\omega\tau}$ due to the screening of the original field by the field of produced pairs.

$$\phi(p_\perp) \propto S_\perp e^{-\frac{4\pi p_-}{\omega}} = S_\perp e^{-\frac{p_-}{T}}$$

Momentum conservation implies

$$T \approx \frac{1}{2\sqrt{2\pi Q_s}}$$

Phenomenology
**k_T-factorization**

- **kT-factorization** assumes that \( qq \) production process can be factorized out from the wave functions of proton and nucleus.

\[
\frac{d\sigma}{d^2 k dy_1 dy_2} = \int d^2 q_1 \int d^2 q_2 \phi_p(q_1, y_1) A_{gg}(s, t, u, q_1, q_2) \phi_A(q_2, y_2)
\]

- Cross section is much easier to calculate.

- It was proved for single inclusive gluon production in \( pA \) (Kovchegov, K.T. hep-ph/0111362);

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Levin, Ryskin, Shabelsky, Shuvaev, Catani, Ciafaloni, Hautmann, Collins, Ellis All:1991
How good is $k_T$-factorization?

Fujii, Gelis, Venugopalan hep-ph/0504047

Ratio of the exact result to the $k_T$-factorization:
Open charm yield at RHIC

A model based on $k_T$-factorization

Suppression onsets when $Q_s > m$. Production pattern of light and heavy quarks becomes the same.
Data from PHENIX

\[ R_{pA} = \frac{d\sigma_{pA}}{dyd^2k} \frac{1}{A} \frac{d\sigma_{pp}}{dyd^2k} \]

Both suppression at forward rapidity and enhancement at backward one are consistent with CGC.

Presented by R. Averbeck at RHIC & AGC Users’ Meeting
$J/\psi$ production

K.T. and D. Kharzeev, hep-ph/0510358
Time scales for $J/\psi$ production

A pre-hadron $cc$ pair is produced over time

$$\tau_P \approx \frac{E_g}{M_\psi^2} = 7 \, e^y \, \text{fm}$$

$J/\psi$ wave function is formed over time

$$\tau_F \approx \frac{2}{M_{\psi'} - M_\psi} \frac{E_g}{M_\psi} = 42 \, e^y \, \text{fm}$$

Hierarchy of scales required for the dipole model: $T_F \gg T_P \gg T_{\text{int}}$
At $y \approx 1$ cc is produced coherently over entire nucleus and $J/\psi$ is formed outside of it.

We expect the same suppression of $R_{dA}$ as for open charm due to the gluon saturation.
At $-1 < y < 0$ cc is produced coherently over a few nucleons.

$J/\psi$ is formed outside the nucleus.

Note additional enhancement by $N_{\text{part}}$

In DIS: Hufner at al
Backward rapidities

- At $-2 < y < -1$ cc is produced incoherently.
- However, $J/\psi$ is still formed outside the nucleus.
- Small nuclear effect due to color transparency of cc pair.
- In the nucleus fragmentation region ($y < -2$) $J/\psi$ is strongly attenuated since it is formed inside the nucleus.
J/ψ at forward rapidities

- cc scattering amplitude projected on the color singlet state (large $N_c$)
  \[ M_1(x, x_0, y, y_0) = \frac{1}{4} \left( \frac{Q_s^2}{4} \right)^2 \left[ (x - y)^2 + (x_0 - y_0)^2 \right] \]

- light cone wave function of a proton
  \[ \Phi(z_1, z_2) = \frac{\alpha_s C_F}{2\pi} \frac{z_1 \cdot z_2}{z_1^2 z_2^2} \]

- light cone wave function of a virtual gluon

- projection on a hadron wave function
  \[ \Psi_\psi \otimes \Psi_{g^*}(r, \alpha = 1/2) = \sqrt{\frac{3\Gamma_{ee} M_\psi}{48\alpha_{em}\pi}} \frac{m^3 r^2}{4} K_2(mr) \]
The final result (in the quasi-classical approximation)

\[
\frac{d\sigma}{dy} = S_A x G(x_1, Q^2) \frac{3 \Gamma_{ee}}{(2\pi)^2 48 \alpha_{em} M_\psi} \int_0^\infty d\zeta \, \zeta^5 \, K_2(\zeta) \left( 1 - e^{-\left( \frac{Q_s(x_2) \zeta}{2 M_\psi} \right)^4} \right) \]

where \( \zeta = r_\perp \cdot M_\psi \)

Effect of small-\( x \) evolution is encoded in the anomalous dimension \( \gamma(1/r_\perp, x_2) \).

Additional assumptions:

- J/\( \psi \) is non-relativistic. Relativistic correction depends on m but not on energy – included in prefactor.

- Only cc interacts with the nucleus. Interaction of g and q, are parametrically same.

- Parametrically small corrections due to the real part and off-diagonal matrix elements are neglected.
Our model vs data (by PHENIX)

Cold nuclear matter absorption at low energies $S=\exp(-\sigma_{\psi N} \rho L)$

Onset of suppression: $Q_s > M_\psi$
Breakdown of $x_F$-scaling

$\sigma_{pA}=A^{\alpha}\sigma_{pp}$

- $\alpha=2/3$ plateau: black disk regime.
- $x_F$-scaling at lower energies is due to slow energy dependence of $Q_s$. 

$\sigma_p$!
Summary

- High parton density effects are important in pA and AA collisions at RHIC and higher energies.

- We can use interplay of a semi-hard scale $Q_s(A,s)$ and the quark mass $m$ to shed light on the open and hidden heavy quark production mechanisms.

- Nuclear effects in $J/\psi$ production are very interesting and deserve further study.