Quarkonium correlators and spectral functions from anisotropic lattices

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References

Work in collaboration with A. Jakovac (Budapest, Tech. U.), P. Petreczky (Brookhaven) and K. Petrov (Bohr Inst).

- hep-lat/0603005
Outline

1. Introduction
   - Meson Correlators and Spectral Functions
   - Reconstruction of the Spectral Function
   - Simulation parameters, lattices

2. Charmonium
   - Zero Temperature
   - Finite Temperature

3. Bottomonium

4. Summary
Point meson operator

\[ J_H(t, x) = \bar{q}(t, x) \Gamma_H q(t, x), \]

where \( \Gamma_H = 1, \gamma_5, \gamma_\mu, \gamma_5 \gamma_\mu, \gamma_\mu \gamma_\nu. \)

Meson states in different channels:

<table>
<thead>
<tr>
<th>( \Gamma )</th>
<th>( ^2S+1L_J )</th>
<th>( J^{PC} )</th>
<th>( c\bar{c} ) (n=1)</th>
<th>( c\bar{c} ) (n=2)</th>
<th>( b\bar{b} ) (n=1)</th>
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</tr>
</thead>
<tbody>
<tr>
<td>( \gamma_5 )</td>
<td>( ^1S_0 )</td>
<td>0--</td>
<td>( \eta_c )</td>
<td>( \eta_c' )</td>
<td>( \eta_b )</td>
<td>( \eta_b' )</td>
</tr>
<tr>
<td>( \gamma_s )</td>
<td>( ^1S_1 )</td>
<td>0--</td>
<td>( \gamma / \psi )</td>
<td>( h_c )</td>
<td>( \Upsilon(1S) )</td>
<td>( h_b )</td>
</tr>
<tr>
<td>( \gamma_s \gamma_s' )</td>
<td>( ^1P_1 )</td>
<td>0--</td>
<td>( \chi_{c0} )</td>
<td>( \chi_{c1} )</td>
<td>( \omega_{bc}(1P) )</td>
<td>( \chi_{b0}(2P) )</td>
</tr>
<tr>
<td>1</td>
<td>0--</td>
<td>( \chi_{c2} )</td>
<td>( \chi_{b1}(1P) )</td>
<td>( \chi_{b2}(1P) )</td>
<td></td>
<td></td>
</tr>
<tr>
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<tr>
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<td>0(^++)</td>
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The spectral function
\[
\sigma_H(p_0, \vec{p}) = \frac{1}{2\pi} (D_H^>(p_0, \vec{p}) - D_H^<(p_0, \vec{p})) = \frac{1}{\pi} \text{Im} D_H^R(p_0, \vec{p})
\]

\[
D_H^>(<)(p_0, \vec{p}) = \int \frac{d^4p}{(2\pi)^4} e^{i\vec{p} \cdot \vec{x}} D_H^>(<)(x_0, \vec{x})
\]

\[
D_H^>(x_0, \vec{x}) = \langle J_H(x_0, \vec{x}), J_H(0, \vec{0}) \rangle
\]

\[
D_H^<(x_0, \vec{x}) = \langle J_H(0, \vec{0}), J_H(x_0, \vec{x}) \rangle, x_0 > 0
\]

The Euclidean propagator
\[
G_H(\tau, \vec{p}) = \int d^3xe^{i\vec{p} \cdot \vec{x}} \langle T_\tau J_H(\tau, \vec{x}) J_H(0, \vec{0}) \rangle
\]
is related to the spectral function through the integral representation
\[
G(\tau, \vec{p}) = \int_0^\infty d\omega \sigma K(\omega, \tau), \quad K(\omega, \tau) = \frac{\cosh(\omega(\tau - 1/2T))}{\sinh(\omega/2T)}.
\]
The spectral function

\[ \sigma_H(p_0, \vec{p}) = \frac{1}{2\pi} (D^>_H(p_0, \vec{p}) - D^<_H(p_0, \vec{p})) = \frac{1}{\pi} \text{Im}D^R_H(p_0, \vec{p}) \]

\[ D^>_H(\langle \rangle)(p_0, \vec{p}) = \int \frac{d^4p}{(2\pi)^4} e^{ip.x} D_H^>(\langle \rangle)(x_0, \vec{x}) \quad (1) \]

\[ D_H^>(x_0, \vec{x}) = \langle J_H(x_0, \vec{x}), J_H(0, \vec{0}) \rangle \]

\[ D_H^<(x_0, \vec{x}) = \langle J_H(0, \vec{0}), J_H(x_0, \vec{x}) \rangle, x_0 > 0 \quad (2) \]

The Euclidean propagator

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Reconstruction of the Spectral Function

\[ G(\tau, \vec{p}) = \int_0^\infty d\omega \sigma(\omega, T) K(\omega, \tau, T) \]

- \(O(10)\) data and \(O(100)\) degrees of freedom to reconstruct.
- Bayesian technique: find \(\sigma(\omega, T)\) that maximizes \(P[\sigma|DH]\).
  - \(D\) data
  - \(H\) prior knowledge: \(\sigma(\omega, T) > 0\)

Maximum Entropy Method: Asakawa, Hatsuda, Nakahara, PRD 60 (99) 091503, Prog. Part. Nucl. Phys. 46 (01) 459

\[ P[\sigma|DH] = \exp\left(-\frac{1}{2} \chi^2 + \alpha S\right) \]

Shannon-Janes entropy: \(S = \int d\omega \left[ \sigma(\omega) - m(\omega) - \sigma(\omega) \ln\left(\frac{\sigma(\omega)}{m(\omega)}\right)\right]\),

\(m(\omega)\) - the default model, \(m(\omega \gg \Lambda_{QCD}) = m_0 \omega^2\) - perturbation theory.
Reconstruction of the Spectral Function

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Anisotropic lattice $\xi = a_s/a_\tau = 2$ and 4.

Standard Wilson action in the gauge sector and the anisotropic clover improved (Fermilab) action for heavy fermions. Quenched approximation. Sommer scale to fix the physical units.
<table>
<thead>
<tr>
<th>$\beta$</th>
<th>5.7</th>
<th>5.9</th>
<th>6.1</th>
<th>6.1</th>
<th>6.5</th>
</tr>
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<tbody>
<tr>
<td>$N_s^2 \times N_t$</td>
<td>$8^3 \times 64$</td>
<td>$16^3 \times 64$</td>
<td>$16^3 \times 64$</td>
<td>$16^3 \times 96$</td>
<td>$24^3 \times 160$</td>
</tr>
<tr>
<td>$(\xi, \xi_0)$</td>
<td>$(2,1.655)$</td>
<td>$(2,1.691)$</td>
<td>$(2,1.718)$</td>
<td>$(4,3.211)$</td>
<td>$(4,3.3166)$</td>
</tr>
<tr>
<td>$r_0/a_s$</td>
<td>2.414(8)</td>
<td>3.690(11)</td>
<td>5.207(29)</td>
<td>5.189(21)</td>
<td>8.96(4)</td>
</tr>
<tr>
<td>$L_s$ [fm]</td>
<td>1.66</td>
<td>2.17</td>
<td>1.54</td>
<td>1.54</td>
<td>1.34</td>
</tr>
<tr>
<td>configs</td>
<td>2000</td>
<td>1560</td>
<td>930</td>
<td>500</td>
<td>160</td>
</tr>
</tbody>
</table>

Table: Simulation parameters for charmonium at zero temperature.
Charmonium: $T = 0$

Figure: Charmonium spectral function in the pseudoscalar channel (left) and the scalar channel (right) at different lattice spacings and zero temperature.
Figure: Charmonium spectral function dependence on the default model (left) and on the maximum time extend (right). Pseudoscalar channel at $a_t^{-1} = 14.12\, GeV$ and zero temperature.
Charmonium: \( T > 0 \)

\[
G_{\text{recon}}(\tau, T) = \int_0^\infty d\omega \sigma(\omega, T = 0) K(\tau, \omega, T)
\]
Figure: The ratio $G(\tau, T)/G_{\text{recon}}(\tau, T)$ of charmonium for pseudoscalar channel at $a_t^{-2} = 8.18$ and 14.11GeV at different temperatures.
Figure: The ratio $G(\tau, T)/G_{\text{recon}}(\tau, T)$ of charmonium for scalar channel at $a_t^{-2} = 8.18$ and $14.11\text{GeV}$ at different temperatures.
Figure: The ratio $G(\tau, T)/G_{\text{recon}}(\tau, T)$ of charmonium for vector channel at $a_t^{-2} = 8.18$ and $14.11\text{GeV}$ at different temperatures.
Figure: Charmonium spectral function in the pseudoscalar channel at $a_t^{-2} = 14.11\text{GeV}$ (left) and the scalar channel (right) at $a_t^{-2} = 8.18\text{GeV}$ at zero and above deconfinement temperatures. For finite temperature scalar channel two different default models are shown.
Bottomonium

**Figure:** Bottomonia correlators (left) and spectral functions (right) in pseudo-scalar channel for different temperatures.
**Figure:** Bottomonia correlators (left) spectral functions (right) in scalar channel for different temperatures.
The $1S (\eta_c, J/\psi)$ charmonium states exist as a resonance in the deconfined phase at $T \approx 1.5 T_c$.

$1P (\chi_{c0}, \chi_{c1})$ charmonium states dissolve at $1.1 T_c$.

Bottomonium states show similar behavior.

The emerging studies of heavy quarkonium properties with dynamical fermions produce consistent results with the quenched approximation.
Summary

- The $1S (\eta_c, J/\psi)$ charmonium states exist as a resonance in the deconfined phase at $T \simeq 1.5 T_c$.
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