Charmonium Hyperfine Splitting Using Quenched Overlap Fermions


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Outline

Motivation

Simulation Details

Comparison with Other Results

Sources of Error
- Discretization errors
- Finite volume errors
- Unquenching and disconnected diagrams

Conclusion and Future Work
Heavy Quarks Using Overlap Fermions

- Overlap fermions have exact chiral symmetry on the lattice.
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- We want to use anisotropic overlap for heavy-light systems
- Here we study a heavy-heavy system using isotropic overlap

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- The road of this lattice calculation has been rather rocky
- Wilson type fermions:
  a term $rD^2$ is added to remove doublers
  introduces $O(a)$ errors
  can be removed by adding $c_{SW} \sigma_{\mu\nu} F^{\mu\nu}$
  hyperfine splitting extremely sensitive to $c_{SW}$
- staggered fermions:
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- NRQCD: $v^6$ terms are important ($v^2$ is only about 0.3 for charm)
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  - staggered fermions: 
    NG/non-NG ambiguity for simulating $\eta_c$
  - NRQCD: $v^6$ terms are important ($v^2$ is only about 0.3 for charm)
  - Overlap fermions, due to chiral symmetry, have no $O(a)$ errors: $c_{SW}$ is “automatically tuned”. No flavor symmetry breaking. Fully relativistic.
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16³ × 72 isotropic lattices, 100 configurations.

Wilson gauge action at β = 6.3345.

We set scale from r₀ and (1P − 1S). For the P states, we use two types of operators, Γ and Δ: for example, for the h_c, we can use \( \bar{\psi} \sigma_{ij} \psi \) or \( \bar{\psi} \gamma_i \Delta_i \psi \).

From r₀ the spacing is \( a = 0.0560 \) fm.


Note, this uses \( r_0 = 0.5 \) fm.

From (1P − 1S), \( a \) is smaller.

Overlap fermions, 26 masses ranging from 0.020-0.85 in lattice units. The bare masses correspond to 70 MeV - 3 GeV. Only 5 of these are used for analysis.
Figure: Effective mass plot for $J/\psi$ and $\eta_c$. The hyperfine splitting, $m_{J/\psi} - m_{\eta_c}$ can be obtained directly from the ratio of the vector to pseudoscalar correlators. The “effective mass” for a such a ratio is also shown for $ma = 0.35$. 

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Figure: We fit the meson masses linearly in quark mass. Fits are shown for $\eta_c$, $J/\psi$ and $h_c$ masses. Charm mass is fixed using $J/\psi$ mass.
Figure: Charmonium spectrum in physical units. Results from both $r_0$ and $1\bar{P} - 1\bar{S}$ scales are shown. Note, for the latter scale, a linear combination of $h_c$ and $\eta_c$ masses, along with the $J/\psi$ mass, is used for input.
Figure: Hyperfine splitting as a function of quark mass, with interpolation shown at $m_c$. The fit form used is $(m_{J/\psi} - m_{\eta_c})a = A/\sqrt{ma} + B/ma$. The interpolations for $m_c$ from the two scales is shown. Note, hyperfine splitting is higher for $(1\bar{P} - 1\bar{S})$ scale in lattice units and also $a$ is smaller, so the result in MeV is considerably higher for that case.
<table>
<thead>
<tr>
<th></th>
<th>(a(r_0))</th>
<th>(a(1\bar{P}–1\bar{S}))</th>
<th>Expt</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(\Gamma)</td>
<td>(\Gamma\Delta)</td>
<td>(\Delta)</td>
</tr>
<tr>
<td>(\eta_c)</td>
<td>3.017(4)</td>
<td>2.977(6)</td>
<td>2.943(9)</td>
</tr>
<tr>
<td>(J/\psi)</td>
<td></td>
<td>2.967(7)</td>
<td>2.980</td>
</tr>
<tr>
<td>(J/\psi - \eta_c)</td>
<td>0.088(4)</td>
<td>0.113(5)</td>
<td>0.144(9)</td>
</tr>
<tr>
<td>(h_c)</td>
<td>3.44(7)</td>
<td>3.53(8)</td>
<td>3.47(12)</td>
</tr>
<tr>
<td>(\chi_{c0})</td>
<td>3.36(5)</td>
<td>3.41(7)</td>
<td>3.39(10)</td>
</tr>
<tr>
<td>(\chi_{c1})</td>
<td>3.39(5)</td>
<td>3.46(7)</td>
<td>3.45(10)</td>
</tr>
<tr>
<td>(m_c a)</td>
<td>0.431</td>
<td>0.343</td>
<td>0.273</td>
</tr>
<tr>
<td>(m_c (GeV))</td>
<td>1.52</td>
<td>1.41</td>
<td>1.30</td>
</tr>
<tr>
<td>(a(fm))</td>
<td>0.0561</td>
<td>0.0480</td>
<td>0.0416</td>
</tr>
</tbody>
</table>

**Table:** Charmonium spectrum (GeV). Only direct statistical errors are included; the statistical error on the lattice spacing \(a\) and systematic errors are not included in this table.

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We extract a $J/\psi - \eta_c$ hyperfine splitting of $97(6)$MeV. This is probably 10% larger than it would be if we had correctly tuned the quark mass, assuming the hyperfine splitting scales like $m_c^{-2}$; with this correction we are in rather good agreement with the result $88(4)$MeV from the closely related overlap fermion action on an isotropic lattice at Wilson $\beta = 6.3[20]$. These two results are significantly larger than typical results using other improved actions such as clover. Assuming that there is very little change in scaling to the continuum this leaves a deviation from experiment of $\sim 30$MeV, which is consistent with the sort of deviations suggested by unquenching[17, 21] and inclusion of disconnected diagrams[22, 23].

**Figure:** J. Dudek, R. Edwards, D. Richards, PRD73, 074507 (2006).
Comparison with Clover Results

<table>
<thead>
<tr>
<th>Columbia</th>
<th>CP-PACS</th>
<th>CP-PACS</th>
<th>QCD-TARO</th>
<th>QCD-TARO</th>
<th>QCD-TARO</th>
</tr>
</thead>
</table>

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Discretization errors

Our result is for a single lattice spacing, no continuum extrapolation is made. However, we only expect discretization errors at a few percent level because:

1. Deviation of the speed of light is only at a few percent level for overlap fermions for the $m_a$ value corresponding to the charm quark.

2. Our scaling study for a different quantity ($m_N/m_V$) shows that the scale dependence is milder for overlap fermions than wilson or staggered.

3. QCD-TARO study finds mild scale dependence for hyperfine splitting, when the Clover co-efficient $c_{SW}$ is tuned non-perturbatively.
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\[ E_P^2 = c^2 \cdot p^2 + m_P^2 \]

Figure: Dispersion relation for overlap fermions. Speed of light \( c \) agrees with 1 within 5% up to \( ma \approx 0.5 \).

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Discretization errors

Figure: Discretization errors for our value of $m_a$ is estimated to be around 5%.
Figure: Overlap fermions seem to have a milder scale dependence than Wilson or staggered fermions.

QCD-TARO result finds that the $a^2$ dependence of hyperfine splitting is extremely mild for non-perturbatively improved clover result.

Their lattice spacing values are 0.0397, 0.0513, 0.0677 and 0.0931 fm respectively.
Using small lattice spacing and an isotropic lattice, we end up with a very small box, only about 0.8-0.9 fm in size. While this should be sufficient for the S states, P states might have substantial errors.

Box size of about 1.2 fm is found to be sufficient. X. Liao and T. Manke, hep-lat/0210030

The fact that the P state operators make a large difference in the final value of the hyperfine splitting when $1\bar{P} - 1\bar{S}$ scale is used, might be an indication of these finite volume errors.
2.5 Finite volume effects

Table 6: Pseudoscalar mass and hyperfine splitting from non-perturbatively improved clover Dirac operator. The lattice spacing is fixed to 0.093 fm ($\beta = 6.0$) and the number of lattice points $L$, hence the physical volume $La$, is varied as indicated in the table. Results, averaged over 100 configurations (190 for $L = 8$), are given in physical units (MeV) with the scale set by $r_0$.

<table>
<thead>
<tr>
<th>$L$</th>
<th>$La$ (fm)</th>
<th>$^1S_0$</th>
<th>$^3S_1$</th>
<th>$^3S_1 - ^1S_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>0.75</td>
<td>2958(10)</td>
<td>3019(12)</td>
<td>61.4(4.4)</td>
</tr>
<tr>
<td>10</td>
<td>0.93</td>
<td>2953(5)</td>
<td>3023(6)</td>
<td>70.6(2.5)</td>
</tr>
<tr>
<td>12</td>
<td>1.12</td>
<td>2957(3)</td>
<td>3032(5)</td>
<td>75.4(2.7)</td>
</tr>
<tr>
<td>14</td>
<td>1.30</td>
<td>2947(3)</td>
<td>3020(4)</td>
<td>72.6(1.9)</td>
</tr>
<tr>
<td>16</td>
<td>1.49</td>
<td>2952(3)</td>
<td>3025(4)</td>
<td>74.9(2.1)</td>
</tr>
<tr>
<td>18</td>
<td>1.68</td>
<td>2949(2)</td>
<td>3021(3)</td>
<td>72.5(1.5)</td>
</tr>
</tbody>
</table>

**Figure:** A finite volume study by QCD-TARO finds hyperfine splitting decreases for a small volume.
Unquenching and disconnected diagrams

- Substantial contribution expected from sea quarks:
  \[ \Delta m(J/\psi - \eta_c) \approx \frac{\alpha_s(m_c)}{m_c^2} |\psi(0)|^2 \]

- Disconnected diagrams contribution:
  Appears to be small, though error bars are large.

![Graph showing vector meson mass versus mass difference](image)

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▶ $O(a)$ discretization errors are absent for overlap fermions, this makes them desirable for studying heavy quarks on the lattice, as no tuning for $c_{SW}$ is needed.

▶ Our results for the charmonium hyperfine splitting using overlap fermions are encouraging. We obtain $88(4)$ MeV for the hyperfine splitting, using the $r_0$ scale.

▶ Study heavy-light systems: anisotropic lattices.

▶ $a_s = 0.175$ fm, $\xi = 2.5$, multi-mass inversion

▶ Two volumes, $12^3 \times 64$ and $16^3 \times 64$. Study volume dependence of spectral weight to tell a scattering state apart from a single particle state.

▶ Since overlap fermions allow us to go to very low pion masses, we can use these lattices to study heavy-light systems, including exotics like $X(3872)$. 
Scattering state and its volume dependence

Normalization condition requires:

\[ | n, \bar{p}, s > \sim \infty \sqrt{\frac{1}{V}} | n, \bar{p}, s > \]

Two point function:

\[ G(t) = \sum_{\bar{x}} < 0 | T(\chi(\bar{x}, t), \bar{\chi}(0)) | 0 > = \sum_{\bar{x}} \sum_n \frac{|< 0 | \chi(0) | n >|^2}{2 M_n V} e^{-M_n t} \]

\[ = \sum_n W_n e^{-M_n t} \sum_{\bar{x}} \Rightarrow V \]

\[ W_n = \frac{|< 0 | \chi(0) | n >|^2}{2 M_n} \]

For one particle bound state
spectral weight (W) will NOT be explicitly dependent on lattice volume
Scattering state and its volume dependence

Normalization condition requires:
\[ | n, \vec{p}, s > \propto \sqrt{\frac{1}{V}} | n, \vec{p}, s > \]

Two point function:
\[ G(t) = \sum_{\vec{x}} < 0 | T(\chi_1(\vec{x}, t)\chi_2(\vec{x}, t)\chi_1(0)\chi_2(0)) | 0 > \]
\[ = \sum_{\vec{x}} \sum_{n_1,n_2} \frac{|0 | \chi_1(0) | n_1 >^2 |0 | \chi_2(0) | n_2 >^2}{2M_{n_1} V \cdot 2M_{n_2} V} e^{-\left(E_{n_1} + E_{n_2}\right)t} \]
\[ = \sum_{n_1,n_2} \frac{W_{n_1} W_{n_2}}{V} e^{-\left(E_{n_1} + E_{n_2}\right)t} \sum_{\vec{x}} \Rightarrow V \]

For two particle scattering state
spectral weight \((W)\) WILL be explicitly dependent on lattice volume
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**Figure:** Effect of inner loop precision on pseudoscalar propagators for heavy quarks. Notice the correlator drops through many orders of magnitude for heavy quarks, and better inner loop precision is needed. We study output of one spin and one color for a single configuration for this illustration. The curves are slightly shifted for clarity.
We use two ways to set the scale - from $r_0$ (using 0.5fm) and from $(1\bar{P} - 1\bar{S})$ splitting in the charmonium system.

Experimental $m_{J/\psi}$ is used to set $m_c$ (in lattice units). This is straightforward when scale from $r_0$ is used.

Use mass of $h_1$, i.e. $^1P_1$ for $\bar{P}$ mass, and $0.25(3m_{J/\psi} + m_{\eta_c})$ for $\bar{S}$ mass.

The spin averaged $(1\bar{P} - 1\bar{S})$ splitting is used because it is expected to be insensitive to lattice artifacts. In this case determination of $a$ and $m_c$ is entangled.

All hadron masses in lattice units are fitted to a straight line, $m_h = A_h x + B_h$, where $x$ is (ma), bare quark mass in lattice units.

Lattice spacing $a$ and bare charm quark mass $m_c$ are two unknowns, $m_{J/\psi}$ and $m(1\bar{P} - 1\bar{S})$ in physical units are two inputs.
<table>
<thead>
<tr>
<th>Valence action</th>
<th>$m_{J/\psi} - m_{\eta_c}$</th>
<th>reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wilson</td>
<td>28(2)</td>
<td>UKQCD ’92</td>
</tr>
<tr>
<td>Clover</td>
<td>52(4)</td>
<td>UKQCD ’92</td>
</tr>
<tr>
<td>Tadpole-improved clover</td>
<td>93(10)</td>
<td>El-Khadra ’93</td>
</tr>
<tr>
<td>Anisotropic, TI clover</td>
<td>77(2)(6)</td>
<td>QCD-TARO ’03</td>
</tr>
<tr>
<td>NRQCD ($v^4$)</td>
<td>90</td>
<td>Klassen ’98</td>
</tr>
<tr>
<td>NRQCD ($v^6$)</td>
<td>71(2)</td>
<td>Chen ’00</td>
</tr>
<tr>
<td>Staggered $\eta_c$(NG)</td>
<td>73(4)/85(8)</td>
<td>CP-PACS ’01</td>
</tr>
<tr>
<td>Staggered $\eta_c$(non-NG)</td>
<td>96(2)</td>
<td>NRQCD ’93</td>
</tr>
<tr>
<td>Overlap</td>
<td>55(5)</td>
<td>Trottier ’97</td>
</tr>
<tr>
<td>2+1 flav.</td>
<td>404(4)</td>
<td>Aoki ’95</td>
</tr>
<tr>
<td>Dynamic TI clover 3 flav.</td>
<td>51(6)</td>
<td>Aoki ’95</td>
</tr>
<tr>
<td>Overlap</td>
<td>88(4)/104(5)</td>
<td>this work</td>
</tr>
<tr>
<td>Dynamic TI clover 2+1 flav.</td>
<td>80</td>
<td>Fermilab ’02</td>
</tr>
<tr>
<td></td>
<td>97(2)</td>
<td>Fermilab ’03</td>
</tr>
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