Spin Correlations and Velocity-Scaling in NRQCD Matrix Elements

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The NRQCD Factorization Approach
(GTB, E. Braaten, G. P. Lepage)

NRQCD

- The effective field theory Nonrelativistic QCD (NRQCD) separates long-distance quarkonium dynamics ($p \lesssim mv$) from short-distance processes ($p \gtrsim m$).
- Physics with $p < \Lambda \sim m$ is reproduced in the effective theory.
- Physics with $p > \Lambda$ is integrated out, but affects the coefficients of local interactions in the effective theory.
- $\Lambda$ is the UV cutoff of the effective theory.
- Leading terms in the heavy-quark velocity $v$ are just the Schrödinger action.

\[ L_0 = \psi^\dagger \left( iD_t + \frac{D^2}{2m} \right) \psi + \chi^\dagger \left( iD_t - \frac{D^2}{2m} \right) \chi \]

\[ D_t = \partial_t + igA_0. \]
\[ D = \partial - igA. \]

- $\psi$ is the Pauli spinor field that annihilates a heavy quark.
- $\chi$ is the Pauli spinor field that creates a heavy antiquark.
• To reproduce QCD completely, we would need an infinite number of interactions. For example, at next-to-leading order in $v^2$ we have

$$
\delta \mathcal{L}_{\text{bilinear}} = \frac{c_1}{8m^3} \left[ \psi^\dagger (D^2)^2 \psi - \chi^\dagger (D^2)^2 \chi \right]
+ \frac{c_2}{8m^2} \left[ \psi^\dagger (D \cdot gE - gE \cdot D) \psi \\
\quad + \chi^\dagger (D \cdot gE - gE \cdot D) \chi \right]
+ \frac{c_3}{8m^2} \left[ \psi^\dagger (iD \times gE - gE \times iD) \cdot \sigma \psi \\
\quad + \chi^\dagger (iD \times gE - gE \times iD) \cdot \sigma \chi \right]
+ \frac{c_4}{2m} \left[ \psi^\dagger (gB \cdot \sigma) \psi - \chi^\dagger (gB \cdot \sigma) \chi \right].
$$

• In practice, work to a given precision in $v$.

• NRQCD predicts $v$-scaling rules for operators and matrix elements.
  - $v^2 \approx 0.3$ for charmonium.
  - $v^2 \approx 0.1$ for bottomonium.
At large $p_T$ (or $p^*$), the inclusive quarkonium production cross section can be written as a sum of products of NRQCD matrix elements and “short-distance” coefficients:

$$\sigma(H) = \sum_n \frac{F_n(\Lambda)}{m^{d_n-4}} \langle 0 | \mathcal{O}_n^H(\Lambda) | 0 \rangle.$$

The $F_n(\Lambda)$ are short-distance coefficients.
- Partonic cross sections to make a $Q\bar{Q}$ pair convolved with parton distributions.
- Calculate as an expansion in $\alpha_s$. 
• Four-fermion operators:

\[ O^H_n = \chi^\dagger \kappa_n \psi \left( \sum_X |H + X \rangle \langle H + X| \right) \psi^\dagger \kappa' \chi. \]

– \( \kappa \) contains Pauli matrices, color matrices, and covariant derivatives.

• The operator matrix elements contain all of the long-distance (nonperturbative physics).

– Probabilities for a \( \bar{Q}Q \) pair to evolve into a heavy-quarkonium.

– They are universal (process independent).

• The sum over operator matrix elements is an expansion in powers of \( v \).
• A similar factorization formula applies to inclusive quarkonium decays:

\[ \Gamma(H \rightarrow LH) = \sum_n \frac{2 \text{Im} f_n(\Lambda)}{m_Q^{d_n - 4}} \langle H | \mathcal{O}_n(\Lambda) | H \rangle, \]

\[ \mathcal{O}_n = \psi^\dagger \kappa_n \chi \chi^\dagger \kappa_n^\prime \psi. \]

• The production matrix elements are the crossed versions of quarkonium decay matrix elements.
  – Only the color-singlet production and decay matrix elements are simply related.

• An important feature of NRQCD factorization:
  Quarkonium decay and production occur through color-octet, as well as color-singlet, \( Q\bar{Q} \) states.

• NRQCD factorization relies on
  – NRQCD,
  – hard-scattering factorization.

• Errors of order
  – \( \Lambda_{QCD}^2 / p_T^2 \) for unpolarized cross sections,
  – \( \Lambda_{QCD} / p_T \) for polarized cross sections.
Polarization of $J/\psi$’s at the Tevatron

- Gluon fragmentation into $J/\psi$ is the dominant production mechanism at large $p_T$ at the Tevatron.

- The gluon produces a $Q\bar{Q}$ pair that evolves nonperturbatively into the $J/\psi$.

- At high $p_T$, gluon is nearly on its mass shell.
  - Hence, it is nearly transversely polarized.
  - The gluon transfers its polarization to the $Q\bar{Q}$ pair.

- Nonrelativistic QCD (NRQCD) velocity-scaling rules predict that, in the evolution, the spin-non-flip interactions dominate over spin-flip interactions.
  - Corrections of order $v^2 \approx 0.3$.

- Hence, the $J/\psi$ is predicted to take on most of the transverse polarization of the gluon (Cho, Wise).
• The CDF data for the polarization parameter $\alpha$ lie significantly below the prediction at the largest $p_T$:

$$d\sigma/d(cos\theta) \propto 1 + \alpha \cos^2 \theta,$$

$$-1 \leq \alpha \leq +1.$$
Lattice Computation of Spin Correlations in NRQCD Color-Octet Matrix Elements

Motivation

• Existing calculations of polarization at the Tevatron assume that $v$-scaling estimates are valid and neglect spin-flip processes in the NRQCD matrix elements.

• $v$ scaling gives the power behavior as $v \to 0$.
  – Nonperturbative coefficients of the powers of $v$ could be large.
  – In making estimates, it is assumed that the coefficients are of order one.

• It has been suggested that the $v$-scaling rules may need to be modified for charmonium (Brambilla, Pineda, Soto, Vairo; Fleming, Rothstein, Leibovich).

• We would like to test the validity of the NRQCD $v$-scaling estimates for the $J/\psi$ production matrix elements.

• But it is not known how to formulate the computation of production matrix elements on a Euclidean lattice.

• Instead, test the $v$-scaling estimates on the corresponding decay matrix elements.
**Lattice NRQCD Action**

- Quark action (Lepage, L. Magnea, Nakhleh, U. Magnea, Hornbostel)

\[ S^{(n)} = a^3 \sum_x \psi^\dagger(x) \psi(x) \]

\[ -a^3 \sum_x \psi^\dagger(x + a\hat{t}) \left( 1 - \frac{aH_0}{2n} \right)^n \left( 1 - \frac{a\delta H}{2} \right) \]

\[ \times U^\dagger_{x,t} \left( 1 - \frac{a\delta H}{2} \right) \left( 1 - \frac{aH_0}{2n} \right)^n \psi(x). \]

- \( H_0 \) is the discrete form of the continuum \( H_0 \):

\[ H_0 = -\frac{\nabla^{(2)}}{2m}. \]

\[ \nabla^{(2)} = \sum_i \nabla^{(+)}_i \nabla^{(-)}_i = \sum_i \nabla^{(-)}_i \nabla^{(+)}. \]

\[ a\nabla^{(+)}_\mu \psi(x) = U_{x,\mu} \psi(x + a\hat{\mu}) - \psi(x). \]

\[ a\nabla^{(-)}_\mu \psi(x) = \psi(x) - U^\dagger_{x-a\hat{\mu},\mu} \psi(x - a\hat{\mu}). \]

- \( \delta H \) is a discrete form of the action of next-to-leading order in \( v^2 \).

  - We use the Lepage et al. forms except for

  \[ \mathbf{D} \cdot g\mathbf{E} - g\mathbf{E} \cdot \mathbf{D}. \]

- There is a similar action for the antiquark.
• \( n \) is taken \( > 1 \) to avoid a lattice-artifact pole that produces instabilities in the heavy-quark propagator.
  
  – We use
    
    \( n = 2 \) for bottomonium,  
    \( n = 3 \) for charmonium.

• We also include improvements of relative-order \( a \) to the lattice action of leading order in \( v^2 \).

• Tadpole improvement is implemented by dividing each link in the action by
  
  \[
  u_0 = \left\langle \frac{1}{3} \text{Tr} U_{\text{plaq}} \right\rangle^{1/4}.
  \]
  
  – Improves the convergence of lattice perturbation theory (more continuum-like).
Matrix-Element Computation

- Create a $Q\bar{Q}$ in a particular spin state and in a singlet color state.
- Propagate forward in Euclidean time according to the lattice NRQCD action in a background gauge field.
- The $Q\bar{Q}$ pair evolves into a quarkonium state whose principal Fock state has the same spin and color.
  - Excited states suppressed by a factor $\exp(-t\Delta E)$.
- Annihilate the $Q\bar{Q}$ pair in a particular spin and color state.
- Multiply by the Hermitian conjugate of this process to obtain the full matrix element.
- Sum over many gauge-field configurations (path-integral over the gauge fields).
Preliminary Results

- Initial computations are on quenched gauge-field configurations (no dynamical fermions).
  - Should reproduce the qualitative features of QCD.

- Preliminary results are based on 400 gauge-field configurations on $12^3 \times 24$ lattices at $\beta = 5.7$.
  - $a = 0.81 \text{ GeV}^{-1}$ for charmonium.
  - $a = 0.73 \text{ GeV}^{-1}$ for bottomonium.

- The quarkonium is well contained in the lattice volume, but the lattice spacing is fairly coarse (requires order-$a$ improvements):
  - $r \approx 1/(mv) \approx 1.2 \text{ GeV}^{-1}$ for charmonium.
  - $r \approx 1/(mv) \approx 0.6 \text{ GeV}^{-1}$ for charmonium.

- Results for the ratio

  $R(S_i, M_i, S_f, M_f) = \frac{\langle 2S_i+1 S_{S_i,M_i} | O_8 \left( 2S_f+1 S_{S_f,M_f} \right) | 2S_i+1 S_{S_i,M_i} \rangle}{\langle 3S_1 | O_1 (3S_1) | 3S_1 \rangle}$. 

  - Average over initial unspecified spins and sum over final unspecified spins.
- \( \Upsilon \) at \( \beta = 5.7 \)

<table>
<thead>
<tr>
<th>Spin Transition</th>
<th>Lattice</th>
<th>( v ) Scaling</th>
</tr>
</thead>
<tbody>
<tr>
<td>singlet ( \rightarrow ) triplet</td>
<td>( 5.5 \times 10^{-3} )</td>
<td>( v^3/(2N_c) \approx 5.3 \times 10^{-3} )</td>
</tr>
<tr>
<td>triplet ( \rightarrow ) singlet</td>
<td>( 1.8 \times 10^{-3} )</td>
<td>( v^3/(2N_c) \approx 5.3 \times 10^{-3} )</td>
</tr>
<tr>
<td>singlet ( \rightarrow ) singlet</td>
<td>( 7 \times 10^{-5} )</td>
<td>( v^4/(2N_c) \approx 1.7 \times 10^{-3} )</td>
</tr>
<tr>
<td>triplet ( \rightarrow ) triplet</td>
<td>( 8 \times 10^{-5} )</td>
<td>( v^4/(2N_c) \approx 1.7 \times 10^{-3} )</td>
</tr>
<tr>
<td>triplet up ( \rightarrow ) triplet up</td>
<td>( 8 \times 10^{-5} )</td>
<td>( v^4/(2N_c) \approx 1.7 \times 10^{-3} )</td>
</tr>
<tr>
<td>triplet up ( \rightarrow ) triplet long.</td>
<td>noise</td>
<td>( v^6/(2N_c) \approx 1.7 \times 10^{-4} )</td>
</tr>
<tr>
<td>triplet up ( \rightarrow ) triplet down</td>
<td>( 3.1 \times 10^{-6} )</td>
<td>( v^6/(2N_c) \approx 1.7 \times 10^{-4} )</td>
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- \( J/\psi \) at \( \beta = 5.7 \)

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<tr>
<td>singlet ( \rightarrow ) triplet</td>
<td>( 3.8 \times 10^{-2} )</td>
<td>( v^3/(2N_c) \approx 2.7 \times 10^{-2} )</td>
</tr>
<tr>
<td>triplet ( \rightarrow ) singlet</td>
<td>( 1.8 \times 10^{-2} )</td>
<td>( v^3/(2N_c) \approx 2.7 \times 10^{-2} )</td>
</tr>
<tr>
<td>singlet ( \rightarrow ) singlet</td>
<td>( 3.9 \times 10^{-4} )</td>
<td>( v^4/(2N_c) \approx 1.5 \times 10^{-2} )</td>
</tr>
<tr>
<td>triplet ( \rightarrow ) triplet</td>
<td>( 9 \times 10^{-4} )</td>
<td>( v^4/(2N_c) \approx 1.5 \times 10^{-2} )</td>
</tr>
<tr>
<td>triplet up ( \rightarrow ) triplet up</td>
<td>( 5 \times 10^{-4} )</td>
<td>( v^4/(2N_c) \approx 1.5 \times 10^{-2} )</td>
</tr>
<tr>
<td>triplet up ( \rightarrow ) triplet long.</td>
<td>( 1.0 \times 10^{-4} )</td>
<td>( v^6/(2N_c) \approx 4.5 \times 10^{-3} )</td>
</tr>
<tr>
<td>triplet up ( \rightarrow ) triplet down</td>
<td>( 3 \times 10^{-4} )</td>
<td>( v^6/(2N_c) \approx 4.5 \times 10^{-3} )</td>
</tr>
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- \( v^2 \approx 0.3 \) for \( J/\psi \); \( v^2 \approx 0.1 \) for \( \Upsilon \).
Discussion

- The hierarchy of $v$ scaling is preserved, but results suggest a smaller expansion parameter ($1/\pi$ for each loop?).

- The triplet $\rightarrow$ singlet transition rate is large.
  - $\eta_c$ production rate at the Tevatron may be comparable to the $J/\psi$ production rate.

- The transverse $\rightarrow$ longitudinal transition is small compared with the transverse $\rightarrow$ transverse transition.
  - The prediction of large transverse polarization at large $p_T$ at the Tevatron is supported.

- Phenomenological production matrix elements give
  - $R(\text{triplet-triplet}) = 5.1-16 \times 10^{-3}$ for the $\Upsilon(1S)$.
  - $R(\text{triplet-triplet}) = 1.9-24 \times 10^{-3}$ for the $J/\psi$.
  - The lattice decay matrix elements seem to be smaller. But decay $\neq$ production, and lattice $\neq$ continuum.
  - Effects from multiple-gluon radiation may decrease the phenomenological values of the color-octet matrix elements.
    Not yet included in $\Upsilon$ analyses.
• According to the heavy-quark spin symmetry, the ratio 
(singlet → triplet)/(triplet → singlet) 
should be approximately 3/1.
  – Corrections are of order $v^2$.
  – Results agree with a 3/1 ratio to within order $v^2$.
• triplet up → triplet down 
should be suppressed as $v^2$ relative to 
triplet up → triplet up.
  – For $J/\psi$, they are comparable.
• At $\beta = 5.7$, the lattice momentum cutoff is 
$\Lambda = \pi/a \approx 4.1$ GeV $> m_c$.
  – When $p > m$, NRQCD is no longer valid and the $v$-scaling rules no longer hold.
  – Spurious power-divergent contributions that go as 
$[\alpha_s(\Lambda)/\pi]^2(\Lambda/m_c)^4$,
$[\alpha_s(\Lambda)/\pi]^2(\Lambda/m_c)^2 \log(\Lambda/m_c)$ 
may be contaminating the $J/\psi$ matrix elements.
  – Work on controlling them is in progress.
  – Preliminary results suggest that lowering $\Lambda$ reduces 
the discrepancies in the $v$ scaling in the $J/\psi$ matrix elements.
Conclusions

• Preliminary lattice calculations support the idea that spin-flip processes are suppressed in charmonium and bottomonium matrix elements.

• The theoretical expectation stands that there should be substantial transverse polarization of $J/\psi$’s at the Tevatron at large $p_T$. 