Resumming the Color-Octet Contribution to 
\[ e^+ e^- \rightarrow J/\psi + X \]

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New Data from BaBar & Belle

($\sqrt{s} \approx 10.6$ GeV)

$\sigma_{\text{tot}} (\text{pb})$

<table>
<thead>
<tr>
<th>BaBar</th>
<th>2.52 ± 0.21 ± 0.21</th>
</tr>
</thead>
<tbody>
<tr>
<td>Belle</td>
<td>1.47 ± 0.10 ± 0.13</td>
</tr>
</tbody>
</table>

Angular distribution

$A(p)$

| $p \lesssim 3.5$ | 0.05 ± 0.22 | 1.5 ± 0.6 |
| $p \gtrsim 3.5$ | 0.7 ± 0.3 | 1.1 ± 0.4 |

$\frac{d\sigma}{dp\,d\cos\theta} = S(p)(1 + A(p)\cos^2\theta)$

Belle

$\frac{\sigma(e^+e^- \rightarrow J/\psi \, c\bar{c})}{\sigma(e^+e^- \rightarrow J/\psi \, X)} = 0.82 \pm 0.15 \pm 0.14$

K. Abe et al. EPS-ID 562
Contributed paper for 2003 Summer Conferences
http://belle.kek.jp
**Theory**

**NRQCD Factorization Formalism**

\[ s = 0.73 \text{ pb} \]

\[ s = 0.20 \text{ pb} \]

\[ s = 0.79 \text{ pb} \]

\[ s = 0.08 \text{ pb} \]

\[ \square_{\text{tot}}^1 = 0.93 \text{ pb} \quad + \quad \square_{\text{tot}}^8 = 0.87 \text{ pb} \quad \rightarrow \quad \square_{\text{tot}} = 1.8 \text{ pb} \]

\[ \frac{\square(e^+e^- \rightarrow J/\Psi c\bar{c})}{\square(e^+e^- \rightarrow J/\Psi X)} = 0.1 \]
Differential Distribution

\[ \frac{d^3 \sigma}{d \phi \, d \Delta \phi \, d \lambda} = \frac{C}{(1 - z)^3} \left( \frac{z}{\Delta \phi} \right) \left( \frac{z(1 - z)}{\phi} \right) \]

\[ z = \frac{E}{E_{\text{beam}}} \]

**BaBar**

S. Baek et al.,
33, 97 (1998)

**Belle**

E. Braaten and Y.Q. Chen,
Angular Distribution

$A(p_y)$

- **Belle**
- **BaBar**

**Color-Singlet**

**Color-Octet**

Recap:

- Total Cross Measurement $\sim 1.5/2.5$ pb
- Color-Singlet under-predicts $\sim 1.0$ pb
- Color-Octet contribution $\sim 0.8$ pb: theoretical cross section is in line with experiment
- Experimental evidence is that Angular Dependence increases to 1 as Psi momentum reaches Max.
- Color-Singlet $\rightarrow -1$ as momentum reaches Max.
- Color-Octet is $\sim 1$ only near Max. momentum
Conclusions?

- Color-Octet is needed to explain the total cross-section
- Data on $A(p)$ suggests the octet contribution is not confined to the endpoint, but is spread over a broad range of momentum
- The $\frac{d}{dz}(1-z)$ does not make sense
- Smeared out by both perturbative and non-perturbative aspects of soft radiation
  BUT how do we treat this?!?!?!
More Trouble?

Compare what happens to the leading singlet and octet contribution as we approach the endpoint:

<table>
<thead>
<tr>
<th></th>
<th>Singlet</th>
<th>Octet</th>
</tr>
</thead>
<tbody>
<tr>
<td>Integrate over full kinematic region</td>
<td>$\nu^3$</td>
<td>$\nu^7$</td>
</tr>
<tr>
<td>Behavior of distribution near endpoint</td>
<td>Constant</td>
<td>$\Box(1 - z)$</td>
</tr>
<tr>
<td>Integrate over endpoint region $\sim \nu^2$</td>
<td>$\nu^5$</td>
<td>$\nu^7$</td>
</tr>
</tbody>
</table>

Power Counting Changes!
Kinematics: CM frame

\[ p_{c\bar{c}}^\mu = M v^\mu + \ell^\mu \]

\[ p_X^\mu = \frac{\sqrt{s}}{2} \left[ \left( 1 - \frac{r}{x} \right) n^\mu + (1 - x) \bar{n}^\mu \right] - \ell^\mu \]

\[ r = 4m_c^2/s \quad x = (E + p)/\sqrt{s} \quad n^\mu = (1,0,0,1) \quad \bar{n}^\mu = (1,0,0,-1) \]

endpoint

\[ x \to 1 \quad p_X^\mu \to \frac{\sqrt{s}}{2} (1 - r) n^\mu \quad p_X^2 \to (1 - r)(s(1 - x) - n \cdot \ell) \sim \sqrt{s} \Box_{QCD} \]

In the endpoint region, \( p_X^\mu \) is a collinear momentum.
NRQCD does not account for this.
Soft Collinear Effective Theory

Effective Field Theory of collinear particles interacting with soft degrees of freedom

"brown muck"

Analogous to HQET: Effective Field Theory of Heavy particles interacting with and soft degrees of freedom

("brown muck")
In the Endpoint Region use SCET for the Fast & Light Particles and NRQCD for the Quarkonium Meson.
NRQCD factorization does not hold near the endpoint: New factorization formula

\[
\frac{d\pi}{dz} = P(r, z) \pi_0 \int_z^1 d\pi S(\pi; \mu) J(\pi - z; \mu)
\]

\[
z = \frac{E_{\pi}}{E_{\max}} \quad P(r, z) = \frac{\sqrt{(1 + r)^2 z^2 - 4r}}{1 - r}
\]

\[
\pi_0(8, ^1S_0) = \frac{32 \pi_0^2 \pi_0^2 e_c^2}{3s^2} \frac{\langle O_8^\uparrow (1S_0) \rangle}{m_c} (1 - r)
\]

Jet function: perturbatively calculable in \( \pi_s \left( \sqrt{sQCD/mc} \right) \)

\[
\rightarrow \quad \pi(\pi - z)
\]

Shape function

\[
S(\pi) = M \frac{1 + r}{1 - r} \frac{\langle 0 | T^A a_\pi^\dagger a_\pi (\pi - in \cdot \hat{D}) a_\pi^\dagger T^A \pi | 0 \rangle}{4m_c \langle O_8^\uparrow (1S_0) \rangle}
\]

Nonperturbative function
Summing Logarithms

At higher orders in the perturbative expansion one encounters

\[ \frac{\log(1 - z)}{1 - z} \]

Integrating within the endpoint \( \rightarrow \log^2 \frac{\Box}{\Box} \)

Sudakov logarithms are summed using the renomalization group in SCET

Summing logs and expanding the jet function to leading order:

\[
\frac{d \Box^{(8,1S_0)}}{dz} = - \int_z^1 \frac{d \Box}{\Box} P[r, z] \Box^{(8,1S_0)}_0 S^{(8,1S_0)} (\Box)
\]

\[
z \frac{d}{dz} \left\{ \Box (\Box - z) \frac{\exp [lg_1 [\Box_s \Box_0 l/(4\Box)] + g_2 [\Box_s \Box_0 l/(4\Box)]]}{\Box [1 - g_1 [\Box_s \Box_0 l/(4\Box)] - \Box_s \Box_0 l/(4\Box) g'_1 [\Box_s \Box_0 l/(4\Box)]]} \right\}
\]
Phenomenology

Model for the shape function

\[ f(\hat{\ell}^+) = \frac{1}{\bar{L}} \frac{a^{ab}}{(ab)} (x - 1)^{ab-1} e^{-a(x-1)} \quad x = \frac{\hat{\ell}^+}{\bar{L}} \]

The Nth moment scales as \( O((mv^2)^N) \)

\[
\begin{align*}
    m_0 &= \int d\hat{\ell}^+ f(\hat{\ell}^+) = 1, \\
    m_1 &= \int d\hat{\ell}^+ \hat{\ell}^+ f(\hat{\ell}^+) = \bar{\square} (b + 1), \\
    m_2 &= \int d\hat{\ell}^+ (\hat{\ell}^+)^2 f(\hat{\ell}^+) = \bar{\square}^2 \left( \frac{b}{a} + (b + 1)^2 \right) \\
\end{align*}
\]

Choose

\[ m_1 = 890 \text{ MeV} \quad m_2 = (985 \text{ MeV})^2 \]
Endpoint: \( z \gtrsim 0.7 \sim 1 - v^2 \)
Comparison to BaBar Data

Endpoint:

\[ z \gtrsim 0.7 \sim 1 - v^2 \]

\[ z = \frac{E_{\square}}{E_{\text{max}}} \]

\[ p \gtrsim \sqrt{(0.7E_{\text{max}})^2 - M_{\square}^2} \gtrsim 2.57 \text{ GeV} \]

Comparison to Belle

Different normalization: all else same

Angular Distribution

$A(p_\psi)$ vs. $p_\psi$

- Belle
- BaBar
Conclusions

- The color-octet contribution at the endpoint is needed to explain $\square_{tot}$

- Need to incorporate collinear physics to make color-octet contribution to $d\square/dp$ sensible

- Angular distribution is consistent with data

- There is still a big problem with $\frac{\square(e^+e^- \rightarrow J/\Psi cc)}{\square(e^+e^- \rightarrow J/\Psi X)}$

- A possible solution was proposed by B.L. Ioffe and D.E. Kharzeev

- My question: does factorization breakdown?!?!